Pearson Edexcel

# Mark Scheme (Results)

## Summer 2022

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 02 Pure Mathematics 2

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

# EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

# General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

## Method mark for solving 3 term quadratic:

### 1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\ldots$

### 2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

### 3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

## Method marks for differentiation and integration:

### 1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

### 2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

## Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

## Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1** | For an attempt to solve **Either** $3-2x=7+x \Rightarrow x=...$ **or** $2x-3=7+x \Rightarrow x=...$ | M1 | 1.1b |
| | **Either** $x=-\frac{4}{3}$ **or** $x=10$ | A1 | 1.1b |
| | For an attempt to solve **Both** $3-2x=7+x \Rightarrow x=...$ **and** $2x-3=7+x \Rightarrow x=...$ | dM1 | 1.1b |
| | For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions | A1 | 1.1b |
| | | **(4)** | |
| **ALT** | **Alternative by squaring:** | | |
| | $(3-2x)^2=(7+x)^2 \Rightarrow 9-12x+4x^2=49+14x+x^2$ | M1 | 1.1b |
| | $3x^2-26x-40=0$ | A1 | 1.1b |
| | $3x^2-26x-40=0 \Rightarrow x=...$ | dM1 | 1.1b |
| | For **both** $x=-\frac{4}{3}$ **and** $x=10$ with no extra solutions | A1 | 1.1b |
| | | **(4 marks)** | |

**Notes:**

**Note this question requires working to be shown not <u>just answers written down</u>. But correct equations seen followed by the correct answers can score full marks.**

**M1**: Attempts to solve either correct equation.
Allow equivalent equations e.g. $3-2x=-7-x \Rightarrow x=...$

**A1**: One correct solution. Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not e.g. -1.33

**dM1**: Attempts to solve both correct equations.
Allow equivalent equations e.g. $3-2x=-7-x \Rightarrow x=...$ **Depends on the first method mark.**

**A1**: For both $x=-\frac{4}{3}$ and $x=10$ with no extra solutions and neither clearly rejected but ignore any attempts to find the *y* coordinates whether correct or otherwise and ignore reference to e.g. $x = -7$ (from where $y = 7 + x$ intersects the *x*-axis) or $x=1.5$ (from finding the value of *x* at the vertex) as "extras". Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not rounded e.g. -1.33

Isw if necessary e.g. ignore subsequent attempts to put the values in an inequality e.g. $-\frac{4}{3}<x<10$

But if e.g. $x=-\frac{4}{3}$ is obtained and a candidate states $x=\left|-\frac{4}{3}\right|$ then score A0

**Alternative solution via squaring**

**M1**: Attempts to square both sides. Condone poor squaring e.g. $(3-2x)^2 = 9 \pm 4x^2$ or $9 \pm 2x^2$

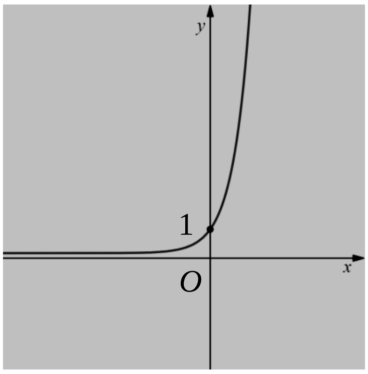
**A1**: Correct quadratic equation $3x^2 - 26x - 40 = 0$. The "= 0" may be implied by their attempt to solve. Terms must be collected but not necessarily all on one side so allow e.g. $3x^2 - 26x = 40$

**dM1**: Correct attempt to solve a **3 term** quadratic. See general guidance for solving a quadratic equation. The roots can be written down from a calculator so the method may be implied by their values. **Depends on the first method mark**.

**A1**: For both $x = -\frac{4}{3}$ and $x = 10$ with no extra solutions and neither clearly rejected but ignore any attempts to find the *y* coordinates and do not count e.g. $x = -7$ (from where $y = 7 + x$ intersects the *x*-axis) or $x = 1.5$ (from finding the value of *x* at the vertex) as "extras". Allow exact equivalents for $-\frac{4}{3}$ e.g. $-1\frac{1}{3}$ or $-1.\dot{3}$ but not e.g. -1.33

Isw if necessary e.g. ignore subsequent attempts to put the values in an inequality e.g. $-\frac{4}{3} < x < 10$

But if e.g. $x = -\frac{4}{3}$ is obtained and a candidate states $x = \left|-\frac{4}{3}\right|$ then score A0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** |  Correct shape or correct intercept – see notes | B1 | 1.2 |
| | Fully correct – see notes | B1 | 1.1b |
| | | **(2)** | |
| **(b)** | $4^x = 100 \Rightarrow x = \log_4 100$<br>or<br>e.g. $x\log 4 = \log 100 \Rightarrow x = \dfrac{\log 100}{\log 4}$ | M1 | 1.1b |
| | $\Rightarrow (x =)$ awrt 3.32 | A1 | 1.1b |
| | | **(2)** | |
| | | | **(4 marks)** |

**Notes:**

**Note that B0B1 is not possible in part (a)**

(a) **Axes do not need to be labelled. No sketch is no marks.**

**B1**: Correct shape or correct intercept.

**Shape**: A positive exponential curve in quadrants 1 and 2 only, passing through a point on the positive $y$-axis. Must "level out" in quadrant 2 but not necessarily asymptotic to the $x$-axis and allow if the curve bends up slightly for $x < 0$ but do not allow a clear "U" shape. It must not clearly "stop" on the $x$-axis to the left of the $y$-axis.

**OR**

**Intercept**: The intercept can be marked as 1 or (0, 1) or $y = 1$ or (1, 0) as long as it is in the correct place. May also be seen away from the sketch but must be seen as (0, 1) or possibly these coordinates in a table but it must correspond to the sketch. If there is any ambiguity, the sketch takes precedence.

**B1**: Fully correct.

**Shape**: A positive exponential curve in quadrants 1 and 2 only, passing through a point on the positive $y$-axis. The curve must appear to be asymptotic to the $x$-axis **and it must level out at least half way below the intercept**. Allow if the curve bends up slightly for $x < 0$ but do not allow a clear "U" shape. The curve must not bend back on itself on the rhs of the $y$-axis. There must be no suggestion that the curve approaches another horizontal asymptote other than the $x$-axis e.g. a horizontal dotted line that the curve approaches.

**AND**

**Intercept**: As above

See practice items and below for some examples:

(b)

**M1**: Uses logs in an attempt to solve the equation. E.g. takes log base 4 and obtains $x = \log_4 100$

Alternatively takes logs (any base) to obtain $x\log 4 = \log 100$ and proceeds to $x = \dfrac{\log 100}{\log 4}$

Allow if this subsequently becomes e.g. log 25 as long as $\dfrac{\log 100}{\log 4}$ is seen **but**

$x\log 4 = \log 100 \Rightarrow x = \log 25$ or $x\log 4 = \log 100 \Rightarrow x = \log 100 - \log 4$ scores M0

**A1**: awrt 3.32 . A correct answer only of awrt 3.32 scores M1A1

Note that a common incorrect answer is $x$ = 3.218875… and comes from ln 25 or ln 100 – ln4 and unless $x = \dfrac{\ln 100}{\ln 4}$ is seen previously, this scores M0A0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(a)(i) | $a_1 = 3,\ a_2 = 5,\ a_3 = 3\ \ldots$ | B1 | 1.1b |
| (ii) | 2 | B1 | 1.1b |
| | | **(2)** | |
| (b) | $\sum_{n=1}^{85} a_n = 42\times(3+5)+3$ o.e. | M1 | 3.1a |
| | $= 339$ | A1 | 1.1b |
| | | **(2)** | |
| | | **(4 marks)** | |
| **Notes:** | | | |

(a)(i) Mark (a)(i) and (a)(ii) together.

**B1**: States the values of at least $a_2 = 5$ and $a_3 = 3$. This is sufficient but if more terms are given they must be correct. There is no need to see e.g. $a_2 = \ldots,\ a_3 = \ldots$ just look for values.

Allow an algebraic approach e.g. $a_{n+1} = 8 - a_n,\ a_{n+2} = 8-(8-a_n) = a_n$

A conclusion is **not** needed.

(a)(ii)

**B1**: States that the order of the periodic sequence is 2

Allow "second order", "it repeats every 2 numbers" or equivalent statements that convey the idea of the period being 2.

Note that ±2 is B0

(b)

**M1**: Attempts a **correct** method to find $\sum_{n=1}^{85} a_n$

For example $\sum_{n=1}^{85} a_n = 42\times(3+5)+3$, $\sum_{n=1}^{85} a_n = \frac{84}{2}\times 3 + 42\times 5 + 3$ or $\sum_{n=1}^{85} a_n = 43\times(3+5)-5$

or $\sum_{n=1}^{85} a_n = 43\times 3 + 42\times 5$ or $\sum_{n=1}^{85} a_n = \frac{85}{2}\times 8 - 1$

There may be other methods e.g. "Chunking": 5×(3 + 5) = 40, 40 × 8 = 320, 320 + 3×3 +2×5 = 339

**A1**: 339. Correct answer only scores both marks.

Attempts to use an AP formula score M0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4** | $\dfrac{2(x+h)^2-2x^2}{h}=...$ | M1 | 2.1 |
| | $\dfrac{2(x+h)^2-2x^2}{h}=\dfrac{4xh+2h^2}{h}$ | A1 | 1.1b |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}(4x+2h)=4x$* | A1* | 2.5 |
| | | **(3)** | |
| | | | **(3 marks)** |
| **Notes:** | | | |

Throughout the question allow the use of $\delta x$ for $h$ or any other letter e.g. $\alpha$ if used consistently.
If $\delta x$ is used then you can condone e.g. $\delta^2 x$ for $\delta x^2$ as well as condoning e.g. poorly formed $\delta$'s

**M1**: Begins the process by writing down the gradient of the chord and attempts to expand the correct bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$.

Note that $\dfrac{2(x-h)^2-2x^2}{-h}=...$ is also a possible approach.

**A1**: Reaches a correct fraction oe with the $x^2$ terms cancelled out.

E.g. $\dfrac{4xh+2h^2}{h}$, $\dfrac{\cancel{2x^2}+4xh+2h^2-\cancel{2x^2}}{h}$, $4x+2h$

**A1***: Completes the process by applying a limiting argument and deduces that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4x$ with no errors seen. The "$\dfrac{\mathrm{d}y}{\mathrm{d}x}=$" doesn't have to appear but there must be something equivalent e.g. "$\mathrm{f}'(x)=$" or "Gradient =" which can appear anywhere in their working. If $\mathrm{f}'(x)$ is used then there is no requirement to see f ($x$) defined first. Condone e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}\to 4x$ or $\mathrm{f}'(x)\to 4x$.

Condone missing brackets so allow e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2h=4x$

Do not allow $h$ = 0 if there is never a reference to h→0

e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}=\lim\limits_{h\to 0}4x+2(0)=4x$ is acceptable

but e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{4xh+2h^2}{h}=4x+2h=4x+2(0)=4x$ is not if there is no h→0 seen.

The h→0 does not need to be present throughout the proof e.g. on every line.

They must reach $4x+2h$ at the end and not $\dfrac{4xh+2h^2}{h}$ (without the $h$'s cancelled) to complete the limiting argument.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** | States or uses $h=1.5$ | B1 | 1.1a |
| | Full attempt at the trapezium rule $=\frac{...}{2}\left\{1.63+2.63+2\times\left(2+2.26+2.46\right)\right\}$ | M1 | 1.1b |
| | = awrt 13.3 or $\frac{531}{40}$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)(i)** | $\int_3^9 \log_3\left(2x\right)^{10}\,dx=10\times\text{"13.3"}=\text{awrt}133$ or e.g. $\frac{531}{4}$ | B1ft | 2.2a |
| **(ii)** | $\int_3^9 \log_3 18x\,dx=\int_3^9 \log_3\left(9\times 2x\right)dx=\int_3^9 2+\log_3 2x\,dx$ $=\left[2x\right]_3^9+\int_3^9 \log_3 2x\,dx=18-6+\int_3^9 \log_3 2x\,dx=...$ | M1 | 3.1a |
| | Awrt 25.3 or $\frac{1011}{40}$ | A1ft | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |
| **Notes:** | | | |

(a)

**B1**: States or uses $h=1.5$

**M1**: A full attempt at the trapezium rule.

Look for $\frac{\text{their } h}{2}\left\{1.63+2.63+2\times\left(2+2.26+2.46\right)\right\}$ but condone copying slips

Note that $\frac{\text{their } h}{2}1.63+2.63+2\times\left(2+2.26+2.46\right)$ scores M0 unless the missing brackets are recovered or implied by their answer. You may need to check.

Allow this mark if they add the areas of individual trapezia e.g.

$\frac{\text{their } h}{2}\left\{1.63+2\right\}+\frac{\text{their } h}{2}\left\{2+2.26\right\}+\frac{\text{their } h}{2}\left\{2.26+2.46\right\}+\frac{\text{their } h}{2}\left\{2.46+2.63\right\}$

Condone copying slips but must be a complete method using all the trapezia.

**A1**: awrt 13.3 (Note full accuracy is 13.275) or exact equivalent.

**Note that the calculator answer is 13.324 so you must see correct working to award awrt 13.3**

Use of $h=-1.5$ leading to a negative area can score B1M1A0 but allow full marks if then stated as positive.

(b)(i)

**B1ft**: Deduces that $\int_3^9 \log_3\left(2x\right)^{10}\,dx=10\times\text{"13.3"}=\text{awrt } 133$

FT on their 13.3 look for 3sf accuracy but follow through on e.g. their rounded answer to part (a) so if 13 was their answer to part (a) then allow 130 here **following a correct method**.

A correct method must be seen here but a minimum is e.g. 10 × "13.3" = "133"

Note that $\int_3^9 \log_3\left(2x\right)^{10}\,dx=133.2414316...$ so a correct method must be seen to award marks.

**Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).**

(b)(ii)

**M1**: Shows correct log work to relate the given question to part (a)

Must reach as far as e.g. $[2x]_3^9 + \int_3^9 \log_3 2x \, dx = ...$ with correct use of limits on $[2x]_3^9$ which may be implied or equivalent work e.g. finds the area of the rectangle as $2\times6$

**A1ft**: **Correct working** followed by awrt 25.3 but ft on their 13.3 so allow for 12 + their answer to part (a) **following correct work** as shown.

Note that $\int_3^9 \log_3 18x \, dx = 25.32414...$ **so a correct method must be seen to award marks**.

**Some examples of an acceptable method are:**

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9\times2x) \, dx = \int_3^9 2+\log_3 2x \, dx = 6\times2+\text{"13.3"} = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9\times2x) \, dx = \int_3^9 2+\log_3 2x \, dx = 12+\text{"13.3"} = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9\times2x) \, dx = \int_3^9 2+\log_3 2x \, dx = [2x]_3^9 + \int_3^9 \log_3 2x \, dx = 25.3$$

**BUT just** $12+\text{"13.3"} = 25.3$ **scores M0**

**Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the <u>answer</u> to part (a).**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 6(a) | $(\mathrm{f}'(x)=)4\cos\left(\frac{1}{2}x\right)-3$ | M1<br>A1 | 1.1b<br>1.1b |
| | Sets $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$ | dM1 | 3.1a |
| | $x=14.0$ Cao | A1 | 3.2a |
| | | **(4)** | |
| **(b)** | Explains that $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$<br>**and** the function is continuous | B1 | 2.4 |
| | | **(1)** | |
| **(c)** | Attempts $x_1=5-\dfrac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$<br>(NB f(5) = –1.212… and $\mathrm{f}'(5)=$ –6.204…) | M1 | 1.1b |
| | $x_1=$ awrt 4.80 | A1 | 1.1b |
| | | **(2)** | |
| | | | **(7 marks)** |
| **Notes:** | | | |

(a)

**M1**: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

**A1**: Correct derivative $\mathrm{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $\mathrm{f}'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $\mathrm{f}'(x)=...$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=...$ just look for the expression and the brackets are not required.

**dM1**: For the complete strategy of proceeding to a value for *x*.

Look for

- $\mathrm{f}'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\quad a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}(\pm k)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4...$ or 11.1… (or 82.8… or 637…. or 803 in degrees) would indicate this method.

**A1**: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. 14.011…
Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the *y* coordinate.
**Correct answer with no working scores no marks.**

(b)

**B1**: See scheme. Must be a full reason, (e.g. change of sign and continuous)
Accept equivalent statements for $\mathrm{f}(4)>0$, $\mathrm{f}(5)<0$ e.g. $\mathrm{f}(4)\times\mathrm{f}(5)<0$,"there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because *x* is continuous" or "because the interval is continuous"

(c)

**M1**: Attempts $x_1 = 5 - \frac{\mathrm{f}(5)}{\mathrm{f}'(5)}$ to obtain a value following through on their $\mathrm{f}'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = –5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \frac{\mathrm{f}(5)}{\mathrm{f}'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

**BUT** evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is $4.804624337$ so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

**A1**: $x_1 = \text{awrt } 4.80$ not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{\mathrm{f}(5)}{\mathrm{f}'(5)} = \text{awrt } 4.80$ following a correct derivative scores M1A1

$5 - \frac{\mathrm{f}(5)}{\mathrm{f}'(5)} \neq \text{awrt } 4.80$ with no evidence that $5 - \frac{\mathrm{f}(5)}{\mathrm{f}'(5)}$ was attempted scores M0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7(a)** | $\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$ | B1 | 1.1b |
| | $\left(1-"\frac{9x}{4}"\right)^{\frac{1}{2}}=...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left("-\frac{9x}{4}"\right)^2}{2!}$ or $...+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left("-\frac{9x}{4}"\right)^3}{3!}$ | M1 | 1.1b |
| | $1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{3!}$ | A1 | 1.1b |
| | $\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$ | A1 | 1.1b |
| | | **(4)** | |
| **(b)** | States that the approximation will be an **overestimate** since all terms (after the first one) in the expansion are negative (since $x>0$) | B1 | 3.2b |
| | | **(1)** | |
| | | | **(5 marks)** |
| **Notes:** | | | |

(a)

**B1**: Takes out a factor of 4 and writes $\sqrt{4-9x}=2(1\pm...)^{\frac{1}{2}}$ or $\sqrt{4}(1\pm...)^{\frac{1}{2}}$ or $4^{\frac{1}{2}}(1\pm...)^{\frac{1}{2}}$

**M1**: For an attempt at the binomial expansion of $(1+ax)^{\frac{1}{2}}$ $a\neq 1$ to form term 3 or term 4 with the correct structure. Look for the correct binomial coefficient multiplied by the corresponding power of $x$ e.g.

$\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(...x)^2$ **or** $\frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(...x)^3$ where $...\neq 1$

Condone missing or incorrect brackets around the $x$ terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}$, $\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

**A1**: Correct expression for the expansion of $\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$ e.g.

$$1+\frac{1}{2}\times\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\left(\pm\frac{9x}{4}\right)^2}{2!}+\frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)\times\left(\frac{1}{2}-2\right)\left(-\frac{9x}{4}\right)^3}{3!}$$

which may be left unsimplified as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work. If the 2 outside this expansion is only partially applied to this expansion then score A0 but if it is applied to all terms this A1 can be implied.

**OR** at least 2 correct simplified terms **for the final expansion** from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

**A1**: $\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$ oe and condone e.g. $2+\frac{-9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4},2\frac{1}{4},2.25\right),\left(\frac{81}{64},1\frac{17}{64},1.265625\right),\left(\frac{729}{512},1\frac{217}{512},1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "$x$" is lost then "reappears".

**Direct expansion in (a) can be marked in a similar way:**

$$\sqrt{4-9x}=(4-9x)^{\frac{1}{2}}=4^{\frac{1}{2}}+\left(\frac{1}{2}\right)4^{-\frac{1}{2}}\times(-9x)^1+\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)4^{-\frac{3}{2}}\times\frac{(-9x)^2}{2!}+\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)4^{-\frac{5}{2}}\times\frac{(-9x)^3}{3!}$$

**B1**: For 2 or $\sqrt{4}$ or $4^{\frac{1}{2}}$ as the constant term in the expansion.

**M1**: Correct form for term 3 or term 4.

E.g. $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\times\frac{(...x)^2}{2!}$ or $\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\times\frac{(...x)^3}{3!}$ where $... \neq 1$

Condone missing brackets around the $x$ terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6. Ignore attempts to find more terms.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}$, $\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

**A1**: Correct expansion (unsimplified as above)

**OR** at least 2 correct simplified terms from, $-\frac{9x}{4}$, $-\frac{81x^2}{64}$, $-\frac{729x^3}{512}$

**A1**: $\sqrt{4-9x}=2-\frac{9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$ oe and condone e.g. $2+\frac{-9x}{4}-\frac{81x^2}{64}-\frac{729x^3}{512}$

Allow equivalent mixed numbers and/or decimals for the coefficients e.g.:

$$\left(\frac{9}{4},2\frac{1}{4},2.25\right),\left(\frac{81}{64},1\frac{17}{64},1.265625\right),\left(\frac{729}{512},1\frac{217}{512},1.423828125\right)$$

Ignore any extra terms if found. Allow terms to be "listed" and apply isw once a correct expansion is seen. Allow recovery if applicable e.g. if an "$x$" is lost then "reappears".

(b)

**B1**: States that the approximation will be an **overestimate** due to the fact that all terms (after the first one) in the expansion are negative or equivalent statements e.g.

- Overestimate because the terms are negative
- Overestimate as the terms are being taken away (from 2)

Condone "overestimate as every term is negative"

If you think a response is worthy of credit but are unsure then use Review.

**This mark depends on having obtained an expansion in (a) of the form**

$k-px-qx^2-rx^3 \quad k,p,q,r>0$ but note that if e.g. one of the algebraic terms is zero or was "lost" or there are extra negative terms this mark is still available.

..........................................................................................................................................

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8** | $y=\dfrac{(x-2)(x-4)}{4\sqrt{x}}=\dfrac{x^2-6x+8}{4\sqrt{x}}=\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ | M1<br>A1 | 1.1b<br>1.1b |
| | $\displaystyle\int \frac{1}{4}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}\,\mathrm{d}x=\frac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}\ (+c)$ | dM1<br><br>A1 | 3.1a<br><br>1.1b |
| | Deduces limits of integral are 2 and 4 and applies to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ | M1 | 2.2a |
| | $\left(\dfrac{32}{10}-8+8\right)-\left(\dfrac{2}{5}\sqrt{2}-2\sqrt{2}+4\sqrt{2}\right)=\dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}$<br>Area $R$ $=\dfrac{12}{5}\sqrt{2}-\dfrac{16}{5}$ $\left(\text{or } \dfrac{16}{5}-\dfrac{12}{5}\sqrt{2}\right)$ | A1 | 2.1 |
| | | **(6)** | |
| | | | **(6 marks)** |
| **Notes:** | | | |

**M1**: Correct attempt to write $\dfrac{(x-2)(x-4)}{4\sqrt{x}}$ as a sum of terms with **indices**.

Look for at least two different terms with the correct index e.g. two of $x^{\frac{3}{2}}, x^{\frac{1}{2}}, x^{-\frac{1}{2}}$ which have come from the correct places.

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

**A1**: $\dfrac{1}{4}x^{\frac{3}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$ which can be left unsimplified e.g. $\dfrac{1}{4}x^{2-\frac{1}{2}}-\dfrac{1}{2}x^{\frac{1}{2}}-x^{\frac{1}{2}}+2x^{-\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(x^{\frac{3}{2}}-6x^{\frac{1}{2}}+8x^{-\frac{1}{2}}\right)$

The correct indices may be implied later when e.g. $\sqrt{x}$ becomes $x^{\frac{1}{2}}$ or $\dfrac{1}{\sqrt{x}}$ becomes $x^{-\frac{1}{2}}$

**dM1**: Integrates $x^n \to x^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $x^{\frac{3}{2}}\to x^{\frac{5}{2}},\quad x^{\frac{1}{2}}\to x^{\frac{3}{2}},\quad x^{-\frac{1}{2}}\to x^{\frac{1}{2}}$

It is dependent upon the first M so at least two terms must have had a correct index.

**A1**: $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}(+c)$. Allow unsimplified e.g. $\dfrac{1}{4}\times\dfrac{2}{5}x^{\frac{3}{2}+1}-\dfrac{1}{2}\times\dfrac{2}{3}x^{\frac{1}{2}+1}-\dfrac{2}{3}x^{\frac{1}{2}+1}+2\times 2x^{\frac{1}{2}}$

or as e.g. $\dfrac{1}{4}\left(\dfrac{2}{5}x^{\frac{5}{2}}-4x^{\frac{3}{2}}+16x^{\frac{1}{2}}\right)(+c)$.

**M1**: Substitutes the limits 4 and 2 to their $\dfrac{1}{10}x^{\frac{5}{2}}-x^{\frac{3}{2}}+4x^{\frac{1}{2}}$ and subtracts either way round.

There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\dfrac{1}{10}\times 4^{\frac{5}{2}}-4^{\frac{3}{2}}+4\times 4^{\frac{1}{2}}-\dfrac{1}{10}\times 2^{\frac{5}{2}}-2^{\frac{3}{2}}+4\times 2^{\frac{1}{2}}$

This is an independent mark but the limits must be applied to an expression that is not $y$ so they may even have differentiated.

**A1**: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw

**See overleaf for integration by parts and integration by substitution.**

**Integration by parts:**

| | | |
|---|---|---|
| $\int \frac{(x-2)(x-4)}{4\sqrt{x}}\,dx = \int \frac{1}{4}(x-2)(x-4)x^{-\frac{1}{2}}\,dx = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx$ | M1<br>A1 | 1.1b<br>1.1b |
| $\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int \frac{1}{2}(2x-6)x^{\frac{1}{2}}\,dx} = \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \underline{\int x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\,dx}$<br>$= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} \underline{- \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}}$<br>Or e.g. $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$ | <u>dM1</u><br>A1 | 3.1a<br>1.1b |
| Deduces limits of integral are 2 and 4 and applies to their<br>$\frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{1}{3}x^{\frac{3}{2}}(2x-6) + \frac{4}{15}x^{\frac{5}{2}}$ | M1 | 2.2a |
| $0 - \frac{16}{3} + \frac{128}{15} - \left(0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}\right)$<br>Area $R = \frac{12}{5}\sqrt{2} - \frac{16}{5}$ $\left(\text{or } \frac{16}{5} - \frac{12}{5}\sqrt{2}\right)$ | A1 | 2.1 |
| | **(6)** | |

**Notes:**

**M1**: Applies integration by parts and reaches the form $\alpha(x-2)(x-4)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

oe e.g. $\alpha\left(x^2 - 6x + 8\right)x^{\frac{1}{2}} \pm \int (px+q)x^{\frac{1}{2}}\,dx$ $\alpha, p \neq 0$

**A1**: Correct first application of parts in any form

**dM1**: Attempts their $\int (px+q)x^{\frac{1}{2}}\,dx$ by expanding and integrating or may attempt parts again.

E.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \int \left(2x^{\frac{3}{2}} - 6x^{\frac{1}{2}}\right)dx = \ldots$ or e.g. $\int (2x-6)x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}}(2x-6) - \frac{4}{3}\int x^{\frac{3}{2}}\,dx$

If they expand then at least one term requires $x^n \to x^{n+1}$ or if parts is attempted again, the structure must be correct.

**A1**: Fully correct integration in any form

**M1**: Substitutes the limits 4 and 2 to their $= \frac{1}{2}(x-2)(x-4)x^{\frac{1}{2}} - \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}}$ and subtracts

either way round. There is no requirement to evaluate but 4 and 2 must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $0 - \frac{16}{3} + \frac{128}{15} - 0 + \frac{4}{3}\sqrt{2} + \frac{16}{15}\sqrt{2}$

This is an independent mark but the limits must be applied to a "changed" function.

**A1**: Correct working shown leading to $\frac{12}{5}\sqrt{2} - \frac{16}{5}$ but also allow $\frac{16}{5} - \frac{12}{5}\sqrt{2}$ or exact equivalents

**Attempts at integration by parts "the other way round" should be sent to review.**

**Integration by substitution example:**

| | | |
|---|---|---|
| $u=\sqrt{x}\left(x=u^2\right)\Rightarrow\int\frac{(x-2)(x-4)}{4\sqrt{x}}\mathrm{d}x=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$ <br> $=\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{4u}2u\mathrm{d}u$ | M1<br>A1 | 1.1b<br>1.1b |
| $=\frac{1}{2}\int\left(u^4-6u^2+8\right)\mathrm{d}u=\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$ | dM1<br>A1 | 3.1a<br>1.1b |
| Deduces limits of integral are $\sqrt{2}$ and 2 and applies to their <br> $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ | M1 | 2.2a |
| $\frac{1}{2}\left(\frac{32}{5}-16+16-\left(\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)\right)$ <br> Area $R=\frac{12}{5}\sqrt{2}-\frac{16}{5}\quad\left(\text{or }\frac{16}{5}-\frac{12}{5}\sqrt{2}\right)$ | A1 | 2.1 |
| | **(6)** | |

**Notes:**

**M1**: Applies the substitution e.g. $u=\sqrt{x}$ and attempts $k\int\frac{\left(u^2-2\right)\left(u^2-4\right)}{u}\frac{\mathrm{d}x}{\mathrm{d}u}\mathrm{d}u$

**A1**: Fully correct integral in terms of $u$ in any form e.g. $\frac{1}{2}\int\left(u^2-2\right)\left(u^2-4\right)\mathrm{d}u$

**dM1**: Expands the bracket and integrates $u^n\to u^{n+1}$ for at least 2 correct indices

i.e. at least 2 of $u^4\to u^5,\ u^2\to u^3,\ k\to ku$

**A1**: $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)(+c)$. Allow unsimplified.

**M1**: Substitutes the limits 2 and $\sqrt{2}$ to their $\frac{1}{2}\left(\frac{u^5}{5}-\frac{6u^3}{3}+8u\right)$ and subtracts either way round.

There is no requirement to evaluate but 2 and $\sqrt{2}$ must be substituted either way round with evidence of subtraction, condoning omission of brackets.

E.g. condone $\frac{1}{2}\left(\frac{32}{5}-16+16-\frac{4\sqrt{2}}{5}-4\sqrt{2}+8\sqrt{2}\right)$

Alternatively reverses the substitution and applies the limits 4 and 2 with the same conditions.

**A1**: Correct working shown leading to $\frac{12}{5}\sqrt{2}-\frac{16}{5}$ but also allow $\frac{16}{5}-\frac{12}{5}\sqrt{2}$ or exact equivalents

Award this mark once one of these forms is reached and isw.

**There may be other substitutions seen and the same marking principles apply.**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **9(a)** | Deduces that $A=\pm 50$ **or** $b=\frac{1}{4}$ | B1 | 3.4 |
| | Deduces that $A=\pm 50$ **and** $b=\frac{1}{4}$ | B1 | 3.4 |
| | Uses $t=0, H=1 \Rightarrow \alpha = ...$ E.g. $1="50"\sin(\alpha)^\circ \Rightarrow \alpha = ...$ | M1 | 3.4 |
| | $H=\left\| \pm 50\sin\left(\frac{1}{4}t+1.15\right)^\circ \right\|$ | A1 | 3.3 |
| | | **(4)** | |
| **(b)** | E.g. the minimum height above the ground of the passenger on the original model was 0 m<br>or<br>Adding "*d*" means the passenger does not touch the ground. | B1 | 3.5b |
| | | **(1)** | |
| | | | **(5 marks)** |

**Notes:**

(a) Note that B0B1 is not possible

**B1**: Uses the equation of the given model to deduce that $A=\pm 50$ **or** $b=\frac{1}{4}$ o.e.

May be seen embedded within their equation.

**B1**: Uses the equation of the given model to deduce that $A=\pm 50$ **and** $b=\frac{1}{4}$ o.e.

May be seen embedded within their equation.

**M1**: Uses $t$ = 0 and $H$ = 1 in the equation of the model to find a value for $\alpha$ .

Follow through on their value for *A*. Allow for $\pm 1="50"\sin(\alpha)^\circ \Rightarrow \alpha = ...$ where α is in degrees or radians.

Note that in radians $\sin^{-1}\left(\frac{1}{50}\right) \approx \frac{1}{50}$ (0.0200…) which may appear incorrect but is in fact ok.

Also in degrees a value of e.g. 1.14 (truncated) would indicate the method.

**A1**: Writes down the correct full equation of the model: $H=\left| "\pm" 50\sin\left(\frac{1}{4}t+1.15\right)^\circ \right|$ o.e.

Condone omission of degrees symbol and allow awrt 1.15 for $\alpha$.
Allow if a correct equation is seen anywhere in their solution.

(b)

**B1**: Gives a suitable explanation with no contradictory statements.

Condone "so that pod/capsule/seat/passenger/ferris wheel/it etc. will not hit/touch the ground"

Responses that focus on the starting point of the model are likely to score B0

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 10(a) | Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$<br>Or substitutes $x=\frac{3}{2}$ into $\frac{5-3x}{2x-8}$ | M1 | 3.1a |
| | $\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$ | A1 | 1.1b |
| | | **(2)** | |
| (b) | $\left(\frac{8x+5}{2x+3}=\right)4\pm\frac{...}{2x+3}$ | M1 | 1.1b |
| | $\left(\frac{8x+5}{2x+3}=\right)4-\frac{7}{2x+3}$ | A1 | 2.1 |
| | | **(2)** | |
| (c) | $0\leqslant g^{-1}(x)\leqslant 4$ | B1 | 2.2a |
| | | **(1)** | |
| (d) | Attempts either boundary<br>$f(0)=\frac{8\times 0+5}{2\times 0+3}$ **or** $f(4)=\frac{8\times 4+5}{2\times 4+3}$ | M1 | 3.1a |
| | Attempts both boundaries<br>$f(0)=\frac{8\times 0+5}{2\times 0+3}$ **and** $f(4)=\frac{8\times 4+5}{2\times 4+3}$ | dM1 | 1.1b |
| | Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$ | A1 | 2.1 |
| | | **(3)** | |
| | **Alternative by attempting** $fg^{-1}(x)$ | | |
| | $g^{-1}(x)=\sqrt{16-x}\Rightarrow fg^{-1}(x)=\frac{8\sqrt{16-x}+5}{2\sqrt{16-x}+3}$<br>$fg^{-1}(0)=\frac{8\times 4+5}{2\times 4+3}$ **or** $fg^{-1}(16)=\frac{8\times 0+5}{2\times 0+3}$ | M1 | 3.1a |
| | $fg^{-1}(0)=\frac{8\times 4+5}{2\times 4+3}$ **and** $fg^{-1}(16)=\frac{8\times 0+5}{2\times 0+3}$ | dM1 | 1.1b |
| | Range $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$ | A1 | 2.1 |
| | | **(3)** | |
| | | | **(8 marks)** |
| **Notes:** | | | |

(a)

**M1**: Attempts to solve $\frac{3}{2}=\frac{8x+5}{2x+3}\Rightarrow x=...$ You can condone poor algebra as long as they reach a value for *x*.

Alternatively attempt to substitute $x=\frac{3}{2}$ into $f^{-1}(x)=\frac{\pm 5\pm 3x}{\pm 2x\pm 8}$ or equivalent (may be in terms of *y*). Note that attempts to find e.g. $f'(x)$ or $\frac{1}{f(x)}$ which may be implied by values such as $\frac{6}{17}, \frac{17}{6}, \frac{7}{18}, \frac{18}{7}$ score M0

**A1**: Achieves $\left(f^{-1}\left(\frac{3}{2}\right)=\right)-\frac{1}{10}$. Do not be concerned what they call it, just look for the value e.g. $x=-\frac{1}{10}$ or just $-\frac{1}{10}$ is fine. Correct answer with no (or minimal) working scores both marks.

(b)

**M1**: Attempts to divide $8x+5$ by $2x+3$

Look for $4\pm\frac{...}{2x+3}$ where … is a constant or $8x+5=A(2x+3)+B$ with $A$ or $B$ correct (which may be in a fraction) or in a long division attempt and obtains a quotient of 4 or attempts to express the numerator in terms of the denominator e.g. $\frac{8x+5}{2x+3}=\frac{4(2x+3)+...}{2x+3}$

**A1**: A full and complete method showing $\frac{8x+5}{2x+3}=4-\frac{7}{2x+3}$ or $\frac{8x+5}{2x+3}=4+\frac{-7}{2x+3}$

Also allow for correct values e.g. $A$ = 4, $B$ = –7

Do not isw here e.g. if they obtain $A$ = 4, $B$ = –7 and then write $-7+\frac{4}{2x+3}$ score A0

(c)

**B1**: Deduces $0\leqslant g^{-1}(x)\leqslant 4$ o.e.

E.g. $0\leqslant y\leqslant 4$, $0\leqslant \text{range}\leqslant 4$, $g^{-1}(x)\leqslant 4$ and $g^{-1}(x)\geqslant 0$, $0\leqslant g^{-1}\leqslant 4$, $[0, 4]$

but not e.g. $0\leqslant x\leqslant 4$ , $0\leqslant g(x)\leqslant 4$, (0, 4)

(d)

**M1**: Attempts either boundary. Look for either $f(0)=\frac{8\times 0+5}{2\times 0+3}$ **or** $f(4)=\frac{8\times 4+5}{2\times 4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times 0+3}$ **or** $f(4)=4-\frac{7}{2\times 4+3}$

**dM1**: Attempts both boundaries. Look for $f(0)=\frac{8\times 0+5}{2\times 0+3}$ **and** $f(4)=\frac{8\times 4+5}{2\times 4+3}$

or uses (b) e.g. $f(0)=4-\frac{7}{2\times 0+3}$ **and** $f(4)=4-\frac{7}{2\times 4+3}$

**A1**: Correct answer written in the correct form.

E.g. $\frac{5}{3}\leqslant fg^{-1}(x)\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant \text{range}\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant y\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}$ and $fg^{-1}(x)\geqslant\frac{5}{3}$

$\frac{5}{3}\leqslant fg^{-1}\leqslant\frac{37}{11}$, $fg^{-1}(x)\leqslant\frac{37}{11}\cap fg^{-1}(x)\geqslant\frac{5}{3}$, $\left[\frac{5}{3}, \frac{37}{11}\right]$ but not e.g. $\frac{5}{3}\leqslant x\leqslant\frac{37}{11}$

**PTO for an alternative to (d)**

(d) **Alternative:**

**M1**: Attempts $\mathrm{fg}^{-1}(x)$ and either boundary using $x$ = 0 or $x$ = 16

Look for either $\mathrm{fg}^{-1}(0)=\dfrac{8\times \mathrm{g}^{-1}(0)+5}{2\times \mathrm{g}^{-1}(0)+3}$ **or** $\mathrm{fg}^{-1}(16)=\dfrac{8\times \mathrm{g}^{-1}(16)+5}{2\times \mathrm{g}^{-1}(16)+3}$

Or uses (b) e.g. $\mathrm{fg}^{-1}(0)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(0)+3}$ **or** $\mathrm{fg}^{-1}(16)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(16)+3}$

The attempt at $\mathrm{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$ ) into f

**dM1**: Attempts both boundaries. Look for $\mathrm{fg}^{-1}(0)=\dfrac{8\times \mathrm{g}^{-1}(0)+5}{2\times \mathrm{g}^{-1}(0)+3}$ **and** $\mathrm{fg}^{-1}(16)=\dfrac{8\times \mathrm{g}^{-1}(16)+5}{2\times \mathrm{g}^{-1}(16)+3}$

Or uses (b) e.g. $\mathrm{fg}^{-1}(0)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(0)+3}$ **and** $\mathrm{fg}^{-1}(16)=4-\dfrac{7}{2\times \mathrm{g}^{-1}(16)+3}$

The attempt at $\mathrm{fg}^{-1}(x)$ requires an attempt to substitute $\sqrt{16-x}$ (condone $\pm\sqrt{16-x}$ ) into f

**A1**: Correct answer written in the correct form with exact values.

E.g. $\frac{5}{3}\leqslant \mathrm{fg}^{-1}(x)\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant \text{range}\leqslant\frac{37}{11}$, $\frac{5}{3}\leqslant y\leqslant\frac{37}{11}$, $\mathrm{fg}^{-1}(x)\leqslant\frac{37}{11}$ and $\mathrm{fg}^{-1}(x)\geqslant\frac{5}{3}$

$\frac{5}{3}\leqslant \mathrm{fg}^{-1}\leqslant\frac{37}{11}$, $\mathrm{fg}^{-1}(x)\leqslant\frac{37}{11}\cap\mathrm{fg}^{-1}(x)\geqslant\frac{5}{3}$, $\left[\frac{5}{3},\ \frac{37}{11}\right]$ but not e.g. $\frac{5}{3}\leqslant x\leqslant\frac{37}{11}$

Note that the $\frac{37}{11}$ is sometimes obtained fortuitously from incorrect working so check working carefully.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 11 | $n\left(n^2+5\right)$ | | |
| | Attempts even **or** odd numbers<br>Sets $n=2k$ **or** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$ | M1 | 3.1a |
| | Achieves $2k\left(4k^2+5\right)$ (for $n=2k$ ) and states "even"<br>**Or** achieves $(2k+1)\left(4k^2+4k+6\right)=2(2k+1)\left(2k^2+2k+3\right)$<br>(for $n=2k+1$ ) and states "even"<br>Or e.g.<br>achieves $(2k-1)\left(4k^2-4k+6\right)=2(2k-1)\left(2k^2-2k+3\right)$<br>(for $n=2k-1$ ) and states "even" | A1 | 2.2a |
| | Attempts even **and** odd numbers<br>Sets $n=2k$ **and** $n=2k\pm1$ oe and attempts $n\left(n^2+5\right)$ | dM1 | 2.1 |
| | Achieves $2k\left(4k^2+5\right)$ (for $n=2k$ ) and states "even"<br>**and** achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$<br>(for $n=2k\pm1$ ) and states "even"<br>Correct work and states even for both WITH a final conclusion showing that true for all $n(\in\mathbb{N})$ or e.g. true for all even and odd numbers. | A1 | 2.4 |
| | | **(4)** | |
| | | | **(4 marks)** |

**Notes:**

**M1**: For the key step attempting to find $n\left(n^2+5\right)$ when $n=2k$ **or** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **or** $n=2k\pm7$

Condone the use of e.g. $n=2n$ and $n=2n\pm1$

**A1**: Achieves $2k\left(4k^2+5\right)$ or e.g. $2\left(4k^3+5k\right)$ and deduces that this is even at the appropriate time.

Or achieves $(2k\pm1)\left(4k^2\pm4k+6\right)=2(2k\pm1)\left(2k^2\pm2k+3\right)$ oe e.g. $2\left(4k^3+6k^2+8k+3\right)$ and deduces that this is even.

Note that if the bracket is expanded to e.g. $8k^3+12k^2+16k+6$ then stating "even" is insufficient – they would need to say e.g. even + even + even + even = even or equivalent

Note it is also acceptable to use a divisibility argument e.g. $\dfrac{8k^3+10k}{2}=4k^3+5k$ so $8k^3+10k$ must be even.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

**dM1**: Attempts $n\left(n^2+5\right)$ when $n=2k$ **and** $n=2k\pm1$ or equivalent representation of odd or even e.g. $n=2k+2$ **and** $n=2k\pm7$

**A1**: Correct work and states even for both WITH a final conclusion e.g. so true for all $n(\in\mathbb{N})$.

There should be no errors in the algebra but allow e.g. invisible brackets if they are "recovered".

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A "solution" via just logic.

E.g.

If $n$ is odd, then $n\left(n^2+5\right)$ is odd×(odd + odd) = odd × even = even

If $n$ is even, then $n\left(n^2+5\right)$ is even×(even + odd) = even × odd = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

**OR**

E.g. $n\left(n^2+5\right)=n^3+5n$

If $n$ is odd, then $n^3$ is odd and $5n$ is odd, so $n^3+5n$ is odd + odd = even

If $n$ is even, then $n^3$ is even and $5n$ is even, so $n^3+5n$ is even + even = even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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A solution via contradiction.

**M1 A1**: There exists a number $n$ such that $n\left(n^2+5\right)$ is odd, and so deduces that both $n$ and $n^2+5$ are odd. Note that M1A0 is not possible via this method.

**dM1**: Sets $n^2+5=2k+1$ (for some integer $k$) $\Rightarrow n^2=2k-4=2\left(k-2\right)$ which is even
Must use algebra here for this approach and not a "logic" argument.

**A1**: States that "this is a contradiction as if $n^2$ is even, then $n$ is even" and then concludes so "$n\left(n^2+5\right)$ is even for all $n$."

**Attempts at proof by induction should be sent to review**

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12(a)** | $\mathrm{f}(x)=\dfrac{\mathrm{e}^{3x}}{4x^2+k}\Rightarrow \mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ <br> or <br> $\mathrm{f}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}\Rightarrow \mathrm{f}'(x)=3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ | M1 <br> A1 | 1.1b <br> 1.1b |
| | $\mathrm{f}'(x)=\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ | A1 | 2.1 |
| | | **(3)** | |
| **(b)** | If $y=\mathrm{f}(x)$ has at least one stationary point then <br> $12x^2-8x+3k=0$ has at least one root | B1 | 2.2a |
| | Applies $b^2-4ac\,(\geqslant)\,0$ with $a=12, b=-8, c=3k$ | M1 | 2.1 |
| | $0<k\leqslant\dfrac{4}{9}$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes:**

(a)

**M1**: Attempts the quotient rule to obtain an expression of the form $\dfrac{\alpha\left(4x^2+k\right)\mathrm{e}^{3x}-\beta x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$, $\alpha\beta>0$

condoning bracketing errors/omissions as long as the intention is clear.
If the quotient rule formula is quoted it must be correct.

Condone e.g. $\mathrm{f}'(x)=\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)}$ provided an incorrect formula is not quoted.

May also see product rule applied to $\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}$ to obtain an expression of the form

$\alpha\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}+\beta x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ $\alpha,\beta 0<0$ condoning bracketing errors/omissions as long as the intention is clear. If the product rule formula is quoted it must be correct.

**A1**: Correct differentiation in any form with correct bracketing which may be implied by subsequent work.

**A1**: Obtains $\mathrm{f}'(x)=\left(12x^2-8x+3k\right)\mathrm{g}(x)$ where $\mathrm{g}(x)=\dfrac{\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ **or equivalent**

e.g. $\mathrm{g}(x)=\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$

Allow recovery from "invisible" brackets earlier and apply isw here once a correct answer is seen.
Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\dfrac{\left(4x^2+k\right)3\mathrm{e}^{3x}-8x\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ or $3\mathrm{e}^{3x}\left(4x^2+k\right)^{-1}-8x\mathrm{e}^{3x}\left(4x^2+k\right)^{-2}$ to $\dfrac{\left(12x^2-8x+3k\right)\mathrm{e}^{3x}}{\left(4x^2+k\right)^2}$ for the final mark.

The $"f'(x)="$ must appear at some point but allow e.g. $"\frac{dy}{dx}="$

(b) Note that B0M1A1 is not possible in (b)

**B1**: Deduces that if $y = f(x)$ has at least one stationary point then $12x^2 - 8x + 3k = 0$ has at least one root. There is no requirement to formally state $\frac{e^{3x}}{\left(4x^2+k\right)^2} > 0$

This may be implied by an attempt at $b^2 - 4ac \geqslant 0$ or $b^2 - 4ac > 0$ condoning slips.

**M1**: Attempts $b^2 - 4ac ... 0$ with $a = 12,\ b = -8,\ c = 3k$ where … is e.g. "=", <, >, etc.

Alternatively attempts to complete the square and sets rhs ...0

E.g. $12x^2 - 8x + 3k = 0 \Rightarrow x^2 - \frac{2}{3}x + \frac{1}{4}k = 0 \Rightarrow \left(x - \frac{1}{3}\right)^2 = \frac{1}{9} - \frac{1}{4}k$ leading to $\frac{1}{9} - \frac{1}{4}k \geqslant 0$

**A1**: $0 < k \leqslant \frac{4}{9}$ but condone $k \leqslant \frac{4}{9}$ and condone $0 \leqslant k \leqslant \frac{4}{9}$

Must be in terms of *k* not *x* so do not allow e.g. $0 < x \leqslant \frac{4}{9}$ but condone $\left(0, \frac{4}{9}\right]$ or $\left[0, \frac{4}{9}\right]$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 13(a) | Attempts two of the relevant vectors<br>$\pm\overrightarrow{AB} = \pm(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$<br>$\pm\overrightarrow{AC} = \pm(-20\mathbf{i} + (p+3)\mathbf{j} + 5\mathbf{k})$<br>$\pm\overrightarrow{BC} = \pm(-16\mathbf{i} + (p-4)\mathbf{j} + 4\mathbf{k})$ | M1 | 3.1a |
| | Uses two of the three vectors in such a way as to find the value of *p*. E.g. $p+3 = 5\times 7$ | dM1 | 2.1 |
| | $p = 32$ | A1 | 1.1b |
| | | **(3)** | |
| | **(a) Alternative:** | | |
| | $r_{AB} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k})$ | M1 | 3.1a |
| | $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow \lambda = 5$<br>$4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} + \lambda(-4\mathbf{i} + 7\mathbf{j} + \mathbf{k}) = -16\mathbf{i} + p\mathbf{j} + 10\mathbf{k} \Rightarrow p = 35 - 3$ | dM1 | 2.1 |
| | $p = 32$ | A1 | 1.1b |
| (b) | Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts<br>$\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k}$ | M1 | 3.1a |
| | Correct attempt at $\lambda$ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$<br>$\overrightarrow{CD} = 16\mathbf{i} + (4\lambda - "32")\mathbf{j} + (6\lambda - 10)\mathbf{k}$<br>$\overrightarrow{OA} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$<br>$4\lambda - 32 = -12 \Rightarrow \lambda = ...$ OR $6\lambda - 10 = 20 \Rightarrow \lambda = ...$ | dM1 | 1.1b |
| | $\lvert\overrightarrow{OD}\rvert = 5\times\sqrt{4^2 + 6^2} = 10\sqrt{13}$ | A1 | 1.1b |
| | | **(3)** | |
| | **(b) Alternative:** | | |
| | Deduces that $\overrightarrow{OD} = \lambda\overrightarrow{OB} = 4\lambda\mathbf{j} + 6\lambda\mathbf{k}$ and attempts<br>$\overrightarrow{OD} = \overrightarrow{OC} + \mu\overrightarrow{OA} = -16\mathbf{i} + 32\mathbf{j} + 10\mathbf{k} + \mu(4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ | M1 | 3.1a |
| | Correct attempt at $\lambda$ **or** $\mu$ using the fact that<br>$\lambda\overrightarrow{OB} = \overrightarrow{OC} + \mu\overrightarrow{OA}$<br>E.g. $-16 + 4\mu = 0 \Rightarrow \mu = 4$ | dM1 | 1.1b |
| | $\lvert\overrightarrow{OD}\rvert = 5\times\sqrt{4^2 + 6^2} = 10\sqrt{13}$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes:**

(a)

**M1**: Attempts two of the three relevant vectors by **subtracting** either way around. See scheme.

Allow equivalent work e.g. $\pm\overrightarrow{AB} = \pm(\overrightarrow{OB} + \overrightarrow{AO})$

If no working is shown, method can be implied by 2 correct components.

**dM1**: For the key step in using the fact that if the vectors are parallel, they will be multiples of each other (where the multiple is something other than 1) to find $p$.

E.g. $p+3=5\times 7$, $p-4=\frac{4}{5}(p+3)$, $p-4=4\times 7$

**A1**: $p=32$ (Condone 32**j**)

**For reference,** $\overrightarrow{BC}=4\overrightarrow{AB},\ \overrightarrow{AC}=5\overrightarrow{AB},\ \overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC},\ \overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

**Note that candidates generally only need to use 2 components to find *p* and if the 3rd component has errors but is not used, full marks can be awarded.**

Alternative:

**M1**: Forms the vector equation using $A$ or $B$ as position and $\pm\overrightarrow{AB}$ as the direction

**dM1**: For the key step in using the fact that $C$ lies on the line to find $p$

**A1**: $p=32$ (Condone 32**j**)

**For reference,** $\overrightarrow{BC}=4\overrightarrow{AB},\ \overrightarrow{AC}=5\overrightarrow{AB},\ \overrightarrow{BC}=\frac{4}{5}\overrightarrow{AC},\ \overrightarrow{AC}=\frac{5}{4}\overrightarrow{BC}$

**Note that candidates generally only need to use 2 components to find *p* and if the 3rd component has errors but is not used, full marks can be awarded.**

**There will be other approaches e.g. using "gradients" and "ratios" and the method marks can be implied – if you are unsure if such attempts deserve credit use Review**

(b) Vector approach

**M1**: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts $\overrightarrow{CD}=16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}$

**dM1**: Correct attempt at finding $\lambda$ using the fact that $\overrightarrow{CD}$ is parallel to $\overrightarrow{OA}$

E.g. $16\mathbf{i}+(4\lambda-\text{"}32\text{"})\mathbf{j}+(6\lambda-10)\mathbf{k}=4\alpha\mathbf{i}-3\alpha\mathbf{j}+5\alpha\mathbf{k}\Rightarrow\alpha=4\Rightarrow 4\lambda-\text{"}32\text{"}=-3\times\text{"}4\text{"}\Rightarrow\lambda=...$

**A1**: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

**Alternative:**

**M1**: Deduces that $\overrightarrow{OD}=\lambda\overrightarrow{OB}=4\lambda\mathbf{j}+6\lambda\mathbf{k}$ and attempts

$\overrightarrow{OD}=\overrightarrow{OC}+\mu\overrightarrow{OA}=-16\mathbf{i}+32\mathbf{j}+10\mathbf{k}+\mu(4\mathbf{i}-3\mathbf{j}+5\mathbf{k})$

**dM1**: Correct attempt at finding $\lambda$ or $\mu$ using the fact that $\lambda\overrightarrow{OB}=\overrightarrow{OC}+\mu\overrightarrow{OA}$

E.g. $(-16+4\mu)\mathbf{i}+(\text{"}32\text{"}-3\mu)\mathbf{j}+(10+5\mu)\mathbf{k}=4\lambda\mathbf{j}+6\lambda\mathbf{k}\Rightarrow -16+4\mu=0\Rightarrow\mu=...$

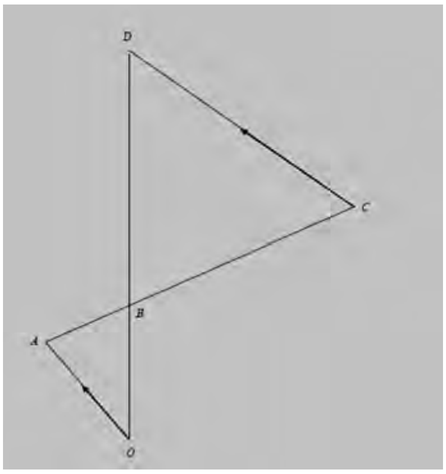
May also solve simultaneously using $y$ and $z$ components to find $\lambda$ or $\mu$

**A1**: $\left|\overrightarrow{OD}\right|=10\sqrt{13}$

Note that the correct vector is 20**j** + 30**k**

**PTO for similar triangle approach**

(b) Similar triangle approach




**M1**: For the key step in recognising that triangle *BCD* and triangle *BAO* are similar with a ratio of lengths of 4:1

**dM1**: States or uses the fact that $\left|\overrightarrow{OD}\right| = 5 \times \left|\overrightarrow{OB}\right|$

Stating this will score M1 dM1 provided there is no evidence of incorrect work

**Note that they may establish this result using the work from (a) but must be used here to score.**

**A1**: $\left|\overrightarrow{OD}\right| = 10\sqrt{13}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14(a)** | $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow A = ..., B = ...$ | M1 | 1.1b |
| | Either $A = 2$ or $B = -1$ | A1 | 1.1b |
| | $\frac{3}{(2x-1)(x+1)} = \frac{2}{2x-1} - \frac{1}{x+1}$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | $\int \frac{1}{V} \mathrm{d}V = \int \frac{3}{(2t-1)(t+1)} \mathrm{d}t$ | B1 | 1.1a |
| | $\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$ | M1 | 3.1a |
| | $\ln V = \ln(2t-1) - \ln(t+1)(+c)$ | A1ft | 1.1b |
| | Substitutes $t = 2, V = 3 \Rightarrow c = (\ln 3)$ | M1 | 3.4 |
| | $\ln V = \ln(2t-1) - \ln(t+1) + \ln 3$<br>$V = \frac{3(2t-1)}{(t+1)}$ * | A1* | 2.1 |
| | | **(5)** | |
| | **(b) Alternative separation of variables:** | | |
| | $\int \frac{1}{3V} \mathrm{d}V = \int \frac{1}{(2t-1)(t+1)} \mathrm{d}t$ | B1 | 1.1a |
| | $\frac{1}{3}\int \frac{2}{2t-1} - \frac{1}{t+1} \mathrm{d}t = ...\ln(2t-1) - ...\ln(t+1)(+c)$ | M1 | 3.1a |
| | $\frac{1}{3}\ln 3V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1)(+c)$ | A1ft | 1.1b |
| | Substitutes $t = 2, V = 3 \Rightarrow c = \left(\frac{1}{3}\ln 3\right)$ | M1 | 3.4 |
| | $\frac{1}{3}\ln V = \frac{1}{3}\ln(2t-1) - \frac{1}{3}\ln(t+1) + \frac{1}{3}\ln 3$<br>$V = \frac{3(2t-1)}{(t+1)}$ * | A1* | 2.1 |
| | | **(5)** | |
| **(c)** | (i) 30 (minutes) | B1 | 3.2a |
| | (ii) 6 ($m^3$) | B1 | 3.4 |
| | | **(2)** | |
| | | | **(10 marks)** |
| **Notes:** | | | |

(a)

**M1**: Correct method of partial fractions leading to values for their $A$ and $B$

E.g. substitution: $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(x+1) + B(2x-1) \Rightarrow A = ..., B = ...$

Or compare coefficients $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = x(A+2B) + A - B \Rightarrow A = ..., B = ...$

Note that $\frac{3}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 3 = A(2x-1) + B(x+1) \Rightarrow A = ..., B = ...$ scores M0

**A1**: Correct value for "*A*" or "*B*"

**A1**: Correct partial fractions not just values for "*A*" and "*B*". $\frac{2}{2x-1}-\frac{1}{x+1}$ or e.g. $\frac{2}{2x-1}+\frac{-1}{x+1}$

Must be seen as **fractions** but if not stated here, allow if the correct fractions appear later.

(b)

**B1**: Separates variables $\int \frac{1}{V}\mathrm{d}V = \int \frac{3}{(2t-1)(t+1)}\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the d*V* and d*t* must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

**M1**: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the (2*t* – 1) and/or the (*t* + 1) for this mark

**A1ft**: Fully correct equation following through their *A* and *B* **only**.

No requirement for +*c* here.

The brackets around the (2*t* – 1) and/or the (*t* + 1) must be seen or implied for this mark

**M1**: Attempts to find "*c*" or e.g. "ln *k*" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

**A1***: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

**Alternative:**

**B1**: Separates variables $\int \frac{1}{3V}\mathrm{d}V = \int \frac{1}{(2t-1)(t+1)}\mathrm{d}t$. May be implied by later work.

Condone omission of the integral signs but the d*V* and d*t* must be in the correct positions if awarding this mark in isolation but they may be implied by subsequent work.

**M1**: Correct attempt at integration of the partial fractions.

Look for $...\ln(2t-1)+...\ln(t+1)$ where … are constants.

Condone missing brackets around the (2*t* – 1) and/or the (*t* + 1) for this mark

**A1ft**: Fully correct equation following through their *A* and *B* **only**.

No requirement for +*c* here.

The brackets around the (2*t* – 1) and/or the (*t* + 1) must be seen or implied for this mark

**M1**: Attempts to find "*c*" or e.g. "ln *k*" using $t=2,\ V=3$ following an attempt at integration.

Condone poor algebra as long as $t=2,\ V=3$ is used to find a value of their constant.

Note that the constant may be found immediately after integrating or e.g. after the ln's have been combined.

**A1***: Correct processing leading to the given answer $V=\frac{3(2t-1)}{(t+1)}$

(Note the working may look like this:

$$\frac{1}{3}\ln 3V=\frac{1}{3}\ln(2t-1)-\frac{1}{3}\ln(t+1)+c,\ \frac{1}{3}\ln 9=\frac{1}{3}\ln(3)-\frac{1}{3}\ln 3+c,\ \ c=\frac{1}{3}\ln 9$$

$$\ln 3V=\ln\frac{9(2t-1)}{(t+1)}\Rightarrow 3V=\frac{9(2t-1)}{(t+1)}\Rightarrow V=\frac{3(2t-1)}{(t+1)}*)$$

**Note that B0M1A1M1A1 is not possible in (b) as the B1 must be implied if all the other marks have been awarded.**

Note also that some candidates may use different variables in (b) e.g.

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3y}{(2x-1)(x+1)}\Rightarrow\int\frac{1}{y}\mathrm{d}y=\int\frac{3}{(2x-1)(x+1)}\mathrm{d}x$ etc. In such cases you should award marks for equivalent work but they must revert to the given variables at the end to score the final mark.

Also if e.g. a "*t*" becomes an "*x*" within their working but is recovered allow full marks.

(c)

**B1**: Deduces 30 minutes. Units not required so just look for 30 but allow equivalents e.g. ½ an hour. If units are given they must be correct so do not allow e.g. 30 hours.

**B1**: Deduces 6 $m^3$. Units not required so just look for 6. Condone $V < 6$ or $V \leq 6$
If units are given they must be correct so do not allow e.g. 6 m.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 15(a) | Uses the common ratio $\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{6\tan\theta}{5+2\sin\theta}$ o.e. | M1 | 3.1a |
| | Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ <br> $(5+2\sin\theta)^2=6\times12\sin\theta$ | dM1 | 1.1b |
| | Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ <br> $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ * | A1* | 2.1 |
| | | **(3)** | |
| **(a) Alt** | **(a) Alternative example:** | | |
| | Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ <br> $\Rightarrow 12\cos\theta\left(\dfrac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$ | M1 | 3.1a |
| | Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ <br> $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$ | dM1 | 1.1b |
| | Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ <br> $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ * | A1* | 2.1 |
| | | **(3)** | |
| (b) | $4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\dfrac{1}{2}\left(,\dfrac{25}{2}\right)$ | M1 | 1.1b |
| | $\theta=\dfrac{5\pi}{6}$ | A1 | 1.2 |
| | | **(2)** | |
| (c) | Attempts a value for either $a$ or $r$ <br> e.g. $a=12\cos\theta=\ 12\times-\dfrac{\sqrt{3}}{2}$ **or** $r=\dfrac{5+2\sin\theta}{12\cos\theta}=\dfrac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$ | M1 | 3.1a |
| | $"a"=-6\sqrt{3}$ **and** $"r"=-\dfrac{1}{\sqrt{3}}$ o.e. | A1 | 1.1b |
| | Uses $S_\infty=\dfrac{a}{1-r}=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ | dM1 | 2.1 |
| | Rationalises denominator <br> $S_\infty=\dfrac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\dfrac{-18}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}$ | ddM1 | 1.1b |
| | $(S_\infty=)9(1-\sqrt{3})$ | A1 | 2.1 |
| | | **(5)** | |
| | | | **(10 marks)** |
| **Notes:** | | | |

(a)

**M1**: For the key step in using the ratio of $\dfrac{a_2}{a_1}=\dfrac{a_3}{a_2}$

**dM1**: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

**A1***: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

**Alternative:**

**M1**: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

**dM1**: Multiplies up and uses $\tan\theta\times\cos\theta = \sin\theta$

**A1***: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

| |
|---|
| Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in $\theta$ using the GP, M1 for applying $\tan\theta\times\cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above |

Example: $u_2 = \dfrac{u_1\times u_3}{u_2} \Rightarrow 5+2\sin\theta = \dfrac{12\cos\theta\times 6\tan\theta}{5+2\sin\theta}$ **M1**

$\Rightarrow (5+2\sin\theta)^2 = 72\sin\theta$ **dM1**

$25+20\sin\theta+4\sin^2\theta = 72\sin\theta$

$\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ * **A1**

(b)

**M1**: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just $x$ from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\dfrac{25}{2}$ does not need to be seen.

**A1**: $\theta = \dfrac{5\pi}{6}$ and no other values unless they are rejected or the $\dfrac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \dfrac{1}{2},\quad \theta = \dfrac{5\pi}{6}$

Do **not** allow 150° for $\dfrac{5\pi}{6}$

**PTO for the notes to part (c)**

(c) Allow full marks in (c) if e.g. $\theta=\frac{\pi}{6}$ is their answer to (b) but $\theta=\frac{5\pi}{6}$ is used here.

or if e.g. $\theta=\frac{5\pi}{6}$ is their answer to (b) but $\theta=\frac{\pi}{6}$ is used here allow the M marks only.

**M1**: For attempting a value (exact or decimal) for either *a* or *r* using **their** $\theta$

E.g. $a=12\cos\theta=\left(12\times-\frac{\sqrt{3}}{2}\right)$ or $r=\frac{5+2\sin\theta}{12\cos\theta}=\left(\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r=\frac{6\tan\theta}{5+2\sin\theta}=\left(\frac{6\times-\frac{1}{\sqrt{3}}}{5+2\times\frac{1}{2}}\right)$

**A1**: Finds both $a=-6\sqrt{3}$ and $r=-\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta=\frac{1}{2},\cos\theta=-\frac{\sqrt{3}}{2}$

and $\tan\theta=-\frac{\sqrt{3}}{3}$ (if required) must have been used.

**dM1**: Uses both **values** of "*a*" and "*r*" with the equation $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ to create an expression

involving surds where *a* and *r* have come from appropriate work and $|r|<1$

Depends on the first method mark.

**ddM1**: Rationalises denominator. The denominator must be of the form $p\pm q\sqrt{3}$ oe e.g. $p+\frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p+q\sqrt{3}}\times\frac{p-q\sqrt{3}}{p-q\sqrt{3}}$ or $\frac{k}{p+\frac{q}{\sqrt{3}}}\times\frac{p-\frac{q}{\sqrt{3}}}{p-\frac{q}{\sqrt{3}}}$ is sufficient.

**A1**: Obtains $(S_\infty=)9\left(1-\sqrt{3}\right)$

Note that full marks are available in (c) for the use of $\theta=150°$
Note also that marks may be implied in (c) by e.g.

$$S_\infty=\frac{a}{1-r}=\frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}}=\frac{144\cos^2\theta}{12\cos\theta-5-2\sin\theta}=\frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}-5-2\sin\frac{5\pi}{6}}$$

$$=\frac{108}{-6-6\sqrt{3}}=\frac{108}{-6-6\sqrt{3}}\times\frac{-6+6\sqrt{3}}{-6+6\sqrt{3}}=\frac{-648+648\sqrt{3}}{-72}=9\left(1-\sqrt{3}\right)$$

Scores M1A1 implied dM1 ddM1 A1

**See next page for some other cases in (c) and how to mark them:**

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **16(a)** | Attempts $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\dfrac{4\sec^2 t\tan t}{2\sec^2 t}\left(=2\tan t\right)$ | M1<br>A1 | 1.1b<br>1.1b |
| | At $t=\dfrac{\pi}{4}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2, x=3, y=7$ | M1 | 2.1 |
| | Attempts equation of normal $y-7=-\dfrac{1}{2}(x-3)$ | M1 | 1.1b |
| | $y=-\dfrac{1}{2}x+\dfrac{17}{2}$ * | A1* | 2.1 |
| | | **(5)** | |
| **(b)** | Attempts to use $\sec^2 t=1+\tan^2 t\Rightarrow\dfrac{y-3}{2}=1+\left(\dfrac{x-1}{2}\right)^2$ | M1 | 3.1a |
| | $\Rightarrow y-3=2+\dfrac{(x-1)^2}{2}\Rightarrow y=\dfrac{1}{2}(x-1)^2+5$ * | A1* | 2.1 |
| | | **(2)** | |
| | **(b) Alternative 1:** | | |
| | $y=\dfrac{1}{2}(x-1)^2+5=\dfrac{1}{2}(2\tan t+1-1)^2+5$<br>$=\dfrac{1}{2}4\tan^2 t+5=2\left(\sec^2 t-1\right)+5$ | M1 | 3.1a |
| | $=2\sec^2 t+3=y$* | A1 | 2.1 |
| | **(b) Alternative 2:** | | |
| | $x=2\tan t+1\Rightarrow t=\tan^{-1}\left(\dfrac{x-1}{2}\right)\Rightarrow y=2\sec^2\left(\tan^{-1}\left(\dfrac{x-1}{2}\right)\right)+3$<br>$\Rightarrow y=2\left(1+\tan^2\left(\tan^{-1}\left(\dfrac{x-1}{2}\right)\right)\right)+3$ | M1 | 3.1a |
| | $\Rightarrow y=2\left(1+\left(\dfrac{x-1}{2}\right)^2\right)+3=\dfrac{1}{2}(x-1)^2+5$* | A1 | 2.1 |
| | **(b) Alternative 3:** | | |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=2\tan t=x-1\Rightarrow y=\int(x-1)\,\mathrm{d}x=\dfrac{x^2}{2}-x+c$<br>$(3,7)\to 7=\dfrac{3^2}{2}-3+c\Rightarrow c=\dfrac{11}{2}$ | M1 | 3.1a |
| | $\dfrac{x^2}{2}-x+\dfrac{11}{2}=\dfrac{1}{2}\left(x^2-2x\right)+\dfrac{11}{2}=\dfrac{1}{2}(x-1)^2-\dfrac{1}{2}+\dfrac{11}{2}=\dfrac{1}{2}(x-1)^2+5$* | A1 | 2.1 |

| | | | |
|---|---|---|---|
| **(c)** | **Attempts the lower limit for *k*:** $\frac{1}{2}(x-1)^2+5=-\frac{1}{2}x+k \Rightarrow x^2-x+(11-2k)=0$ $b^2-4ac=1-4(11-2k)=0 \Rightarrow k=...$ | M1 | 2.1 |
| | $(k=)\frac{43}{8}$ | A1 | 1.1b |
| | **Attempts the upper limit for *k*:** $(x,y)_{t=-\frac{\pi}{4}}: t=-\frac{\pi}{4} \Rightarrow x=2\tan\left(-\frac{\pi}{4}\right)+1=-1,\ y=2\sec^2\left(-\frac{\pi}{4}\right)+3=7$ $(-1,\ 7), y=-\frac{1}{2}x+k \Rightarrow 7=\frac{1}{2}+k \Rightarrow k=...$ | M1 | 2.1 |
| | $(k=)\frac{13}{2}$ | A1 | 1.1b |
| | $\frac{43}{8}<k\leqslant\frac{13}{2}$ | A1 | 2.2a |
| | | **(5)** | |
| | | | **(12 marks)** |
| **Notes:** | | | |

(a) **<u>Must use parametric differentiation to score any marks in this part and not e.g. Cartesian form</u>**

**M1**: For the key step of attempting $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$. There must be some attempt to differentiate both parameters however poor and divide the right way round so using $\frac{dy}{dx}=\frac{y}{x}$ scores M0.

This may be implied by e.g. $\frac{dx}{dt}=2\sec^2 t,\ \frac{dy}{dt}=4\sec^2 t\tan t,\ t=\frac{\pi}{4} \Rightarrow \frac{dx}{dt}=4,\ \frac{dy}{dt}=8 \Rightarrow \frac{dy}{dx}=2$

**A1**: $\frac{dy}{dx}=\frac{4\sec^2 t\tan t}{2\sec^2 t}$. Correct expression in any form. May be implied as above.

Condone the confusion with variables as long as the intention is clear e.g.

$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{4\sec^2 x\tan x}{2\sec^2 x}(=2\tan x)$ and allow subsequent marks if this is interpreted correctly

**M1**: For attempting to find the values of *x*, *y* and the gradient at $t=\frac{\pi}{4}$ **AND** getting at least two correct.

Follow through on their $\frac{dy}{dx}$ so allow for any two of $x=3,\ y=7,\ \frac{dy}{dx}=2\left(\text{or their } \frac{dy}{dx} \text{ at } t=\frac{\pi}{4}\right)$

Note that the $x=3,\ y=7$ may be seen as e.g. (3, 7) on the diagram. There must be a non-trivial $\frac{dy}{dx}$ for this mark e.g. they must have a $\frac{dy}{dx}$ to substitute into.

**M1**: For a correct attempt at the normal equation using their *x* and *y* at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ having made some attempt at $\frac{dy}{dx}$ and all correctly placed.

For attempts using $y = mx + c$ they must reach as far as a value for *c* using their *x* and *y* at $t=\frac{\pi}{4}$ with the negative reciprocal of their $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$ all correctly placed.

**A1***: Proceeds with a clear argument to the given answer with no errors.

(b)

**M1**: Attempts to use $\sec^2 t = 1+\tan^2 t$ oe to obtain an equation involving $y$ and $(x-1)^2$

E.g. as above or e.g. $y = 2\sec^2 t + 3 = 2\left(1+\tan^2 t\right)+3 = 2\left(1+\left(\frac{x-1}{2}\right)^2\right)+3$ for M1 and then

$y = \frac{1}{2}\left(x-1\right)^2 + 5*$ for A1

**A1***: Proceeds with a clear argument to the given answer with no errors

**Alternative 1:**

**M1**: Uses the given result, substitutes for $x$ and attempts to use $\sec^2 t = 1+\tan^2 t$ oe

**A1**: Proceeds with a clear argument to the $y$ parameter and makes a (minimal) conclusion e.g. " = $y$" QED, hence proven etc.

**Alternative 2:**

**M1**: Uses the $x$ parameter to obtain $t$ in terms of arctan, substitutes into $y$ and attempts to use $\sec^2 t = 1+\tan^2 t$ oe

**A1**: Proceeds with a clear argument to the given answer with no errors

**Alternative 3:**

**M1**: Uses $\frac{\mathrm{d}y}{\mathrm{d}x}$ from part (a) to express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $x$, integrates and uses (3, 7) to find "$c$" to reach a Cartesian equation.

**A1**: Proceeds with a clear argument to the given answer with no errors

**Allow the marks for (b) to score anywhere in their solution e.g. if they find the Cartesian equation in part (a)**

(c)

**M1**: A full attempt to find the **lower** limit for $k$.

$$\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k \Rightarrow x^2 - x + \left(11-2k\right) = 0 \Rightarrow b^2 - 4ac = 1 - 4\left(11-2k\right) = 0 \Rightarrow k = ...$$

Score **M1** for setting $\frac{1}{2}(x-1)^2 + 5 = -\frac{1}{2}x + k$, rearranging to 3TQ form and attempts $b^2 - 4ac ... 0$ e.g. $b^2 - 4ac > 0$ or e.g. $b^2 - 4ac < 0$ correctly to find a value for $k$.

**A1**: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

**An alternative method using calculus for lower limit:**

$$y = \frac{1}{2}(x-1)^2 + 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x-1,\ x-1 = -\frac{1}{2} \Rightarrow x = \frac{1}{2}$$

$$x = \frac{1}{2} \Rightarrow y = \frac{1}{2}\left(\frac{1}{2}-1\right)^2 + 5 = \frac{41}{8}$$

$$y = -\frac{1}{2}x + k \Rightarrow \frac{41}{8} = -\frac{1}{4} + k \Rightarrow k = ...$$

Score **M1** for $\frac{\mathrm{d}y}{\mathrm{d}x} =$ "a linear expression in $x$", sets $= -\frac{1}{2}$, solves a linear equation to find $x$ and then substitutes into the given result in (b) to find $y$ and then uses $y = -\frac{1}{2}x + k$ to find a value for $k$. **A1**: $k = \frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k > \frac{43}{8}$, $k < \frac{43}{8}$

**An alternative method using parameters for lower limit:**

$$y=-\frac{1}{2}x+k \Rightarrow 2\sec^2 t+3=-\frac{1}{2}\left(2\tan t+1\right)+k$$

$$\Rightarrow 2\left(1+\tan^2 t\right)+3=-\frac{1}{2}\left(2\tan t+1\right)+k \Rightarrow 2\tan^2 t+\tan t+5.5-k=0$$

$$b^2-4ac=0 \Rightarrow 1-4\times 2\left(5.5-k\right)=0 \Rightarrow k=\frac{43}{8}$$

Score **M1** for substituting parametric form of $x$ and $y$ into $y=-\frac{1}{2}x+k$, uses $\sec^2 t=1+\tan^2 t$ rearranges to 3TQ form and attempts $b^2-4ac...0$ or e.g. $b^2-4ac>0$ or $b^2-4ac<0$ correctly to find a value for $k$.

**A1**: $k=\frac{43}{8}$ oe. Look for this **value** e.g. may appear in an inequality e.g. $k>\frac{43}{8}$, $k<\frac{43}{8}$

**M1**: A full attempt to find the **upper** limit for $k$. This requires an attempt to find the value of $x$ and the value of $y$ using $t=-\frac{\pi}{4}$, the substitution of these values into $y=-\frac{1}{2}x+k$ and solves for $k$.

**A1**: $k=\frac{13}{2}$. Look for this value e.g. may appear in an inequality.

**A1**: Deduces the correct range for $k$: $\frac{43}{8}<k\leqslant\frac{13}{2}$

Allow equivalent notation e.g. $\left(k\leqslant\frac{13}{2} \text{ and } k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cap k>\frac{43}{8}\right)$, $\left(\frac{43}{8}, \frac{13}{2}\right]$

But not e.g. $\left(k\leqslant\frac{13}{2},\ k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2}\cup k>\frac{43}{8}\right)$, $\left(k\leqslant\frac{13}{2} \text{ or } k>\frac{43}{8}\right)$ and do not allow if in terms of $x$.

Allow equivalent exact values for $\frac{43}{8}$, $\frac{13}{2}$

**There may be other methods for finding the upper limit which are valid. If you are in any doubt if a method deserves credit then use Review.**